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Modern Uses of Logic in Law: Quart[illegible]ewsletter of the Electronic Data Retrieval Commit[illegible] of the American Bar Association
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1. APPLICATIONS

a. Mechanization of a Tax Code

Lucien Mehl, in "Automation in the Legal World" (to be published in the Proceedings of the Symposium on the Mechanization of Thought Processes held Nov., 1958, in Teddington, England) points out the need for a mechanization of logic and a new analysis of legal concepts as preliminaries to the use of machines for helping to solve legal problems. Mehl seems well aware of the limitations of machine help: the machine can handle more complex situations than could a human and can perform tasks with great speed, but it can find or deduce only those answers which are present in the data. The machine is no substitute for human judgment.

Mehl suggests a system of analysis of legal situations in terms of binary basic concepts and the use of binary notation to prepare legal problems for machine handling and presents a sample of a tax system which he analyses and renders "machine-able." Suggested changes and corrections have been incorporated in a re-presentation here of this example.

1. A sample tax system may be described in part as follows:

The system is cumulative, so that every transaction involves taxation of the overall value of the product. A principal tax V is levied on any sale of goods made within the country (a domestic sale) by a taxpayer to any person or corporate body, whether or not the receiver of the goods is a taxpayer. The general rate of this tax V is a. Sales of current products (non-luxury products) by retail traders (traders are non-manufacturing sellers) are taxed at the reduced rate. b. If, however, these same retailers sell luxury goods, the tax V is then levied at the increased rate c. Wholesalers, whether manufacturers or traders, are never charged the increased rate. Manufacturers are subject to the principal tax V under the same conditions as are traders except that if they make direct sales to consumers (and are thus manufacturing retailers) they do not qualify for the reduced rate b and they also pay an extra tax X at the rate d in addition to the principal tax V at the general or increased rate. Lastly, exports are tax free; and if goods are exported by a trader, the trader gets a refund of the tax levied at the previous stage.

2. Equations can be written which render explicit and concise the provisions of the presented portion of the tax system. Certain abbreviations (in addition to those noted in 1) will be useful:

C	Seller of current (non-luxury) products	R	Retailer
D	Domestic seller	S	Taxpayer
E	Exporter	SYz	Taxpayer of Tax Y at rate z
FYz	Taxpayer who receives refund of tax Y at rate z	T	Trader
		W	Wholesaler
L	Seller of luxury goods		
M	Manufacturer		

where Y may be V or X, and z may be a, b, c, or d.

In the notation here, the bar indicates negation. Class intersection is expressed by juxtaposition. $\subseteq$ between classes indicates class inclusion; i.e., A $\subseteq$ B indicates that every member of A is a member of B. A = B means that A and B have all the same members, i.e., are identical.

a. WD $\subseteq$ $\overline{\text{SVa}}$ (The class of wholesalers who are domestic sellers is included in the class of taxpayers of tax V at rate a.)

b. RTDC $\subseteq$ $\overline{\text{SVb}}$ (The class of retail traders who are domestic sellers of current products is included in the class of taxpayers of tax V at rate b.)

c. RTDL $\subseteq$ $\overline{\text{SVc}}$ (The class of retail traders who are domestic sellers of luxury goods is included in the class of taxpayers of tax V at rate c.)

d. RMDL $\subseteq$ $\overline{\text{SVc}}$ (The class of retail manufacturers who are domestic sellers of luxury goods is included in the class of taxpayers of tax V at rate c.)

e. RMDC $\subseteq$ $\overline{\text{SVa}}$ (The class of retail manufacturers who are domestic sellers of current products is included in the class of taxpayers of tax V at rate a.)

f. RMD $\subseteq$ $\overline{\text{SXd}}$ (The class of retail manufacturers who are domestic sellers is included in the class of taxpayers of tax X at rate d.)

g. ET $\subseteq$ $\overline{\text{FYz}}$ (The class of exporter traders is included in the class of taxpayers who receive a refund of tax Y at rate z.)

h. E $\subseteq$ $\overline{\overline{\text{S}}}$ (The class of exporters is included in the class of non-taxpayers.)

3. The equations of 2 (hopefully) convey to the reader the provisions of the tax system with more ease and clarity than does the prose statement in 1. But now for ease of machine reading and working, it is useful to identify binary concepts and shift to binary representation. Consider the universe of discourse consisting of those persons subject to the given tax system. The following are binary concepts:

Manufacturer-Trader Domestic seller-Exporter
Wholesaler-Retailer Seller of current products-Seller of luxury goods

Notice that for each person P provided for by the tax system, P is characterized by one or other of the concepts in each of the 4 pairs, and that no person is characterizwd by both concepts of any pair, at least not with respect to a given sale. Now arbitrarily choose the left-hand member of each binary concept as a basic concept. We can then, by taking all possible combinations of affirmations and denials of these concepts, describe 16 (or in general 2^n where n is the number of binary concepts) categories of persons subject to the tax system. The following table gives a binary representation of each of our 8 concepts (4 pairs).

M	0101	0101	0101	0101	T	1010	1010	1010	1010
W	0011	0011	0011	0011	R	1100	1100	1100	1100
D	0000	1111	0000	1111	E	1111	0000	1111	0000
C	0000	0000	1111	1111	L	1111	1111	0000	0000

Each concept is uniquely characterized, as is the class it defines. Note that wherever one of the concepts of a binary concept has a 0, its mate has a 1. For those familiar with truth tables, this characterization procedure corresponds to taking M, W, D, and C as four variables each of which can have either of two values and constructing a 4-column, 16-row truth table, as can be seen by turning this sheet sideways and reading the left side of the above table.

4. Using these binary representations, we can specify very simple procedures (mechanical ones) for finding the intersection of classes, the sum of classes, and for determining whether one class is included in another. These three procedures suffice for handling at least many of the questions which might arise concerning the given tax system. Assuming that the facts have been characterized in the terminology of the tax system, consider such a question as "Is a manufacturer who makes a domestic sale of current products directly to a consumer subject to the extra tax according to the provisions given in 1?" This question can be put thus: RMDC $\subseteq$ SXd? To answer

this question we must determine the class of persons who pay the extra tax (SXd) and whether class RMDC is included in this class.

Consulting the equations of 2, we see that members of RMD pay the extra tax; and since we are concerned only with the provisions in 1 and since no other class is specified as paying the special tax, we know that RMD = SXd. So, using their binary representations, we can find the representation of the intersection of R, M, and D, and then of the intersection of R, M, D, and C; we can then determine whether RMDC is included in RMD. We do this as follows:

R	1100	1100	1100	1100
M	0101	0101	0101	0101
D	0000	1111	0000	1111
RMD	0000	0100	0000	0100

That is, we determine the binary representation of the intersection of two or more classes by aligning their binary representations, then writing a 0 under any column where at least one 0 appears, writing a 1 elsewhere. RMDC is computed as follows:

R	1100	1100	1100	1100
M	0101	0101	0101	0101
D	0000	1111	0000	1111
C	0000	0000	1111	1111
RMDC	0000	0000	0000	0100

(Similarly, to find the binary representation of the sum of two or more classes, align their binary representations and then put a 0 in any column where no 1 appears, writing a 1 elsewhere.) We are now ready to answer our question: RMDC (_ SXd?, or, equivalently, RMDC (_ RMD? Class A is included in class B if there is no place in the binary representation of A such that a 1 appears there and a 0 appears in the corresponding place in the binary representation of B. In terms of our present procedures, align the binary representations of A and B; put a 0 in any column where there is a 1 and A and a 0 in B. Put 1's elsewhere. A is included in B if the resulting row contains only 1's. Thus

RMDC	0000	0000	0000	0100
RMD (SXd)	0000	0100	0000	0100
	1111	1111	1111	1111

So RMDC is included in SXd, and the answer to our question is YES: this manufacturer will have to pay the extra tax. The reader may wish to formulate and answer similar questions using these methods.

5. The equations of 2 could be presented in the following simplified form using u between classes to indicate their sum or union:

a. (WD u RMDC) (_ SVa b. RTDC (_ SVb c. RDL (_ SVc

d. RMD (_ SXd e. TE (_ FVa f. E (_ S

These equations suggest a simpler prose description of the provisions of 1; the reader may find this interesting to construct. This help in improving prose is a useful by-product of the Mehl type of analysis.

Because of the fairly simple nature and small number of concepts involved, the provisions of this example may seem clear even before analysis and the use of a machine may well seem absurd. However, the methods presented here can probably be extended to more complex systems where relations are far from obvious. On the other hand, it is well to note the inherent limitations of this type of analysis and manipulation. It leaves untouched the problems of determining what should classify as a luxury item, what constitutes manufacture of a product, and a host of similar questions of fact and characterizations of facts. But since there are so many things that do require human intervention and judgment, it seems well to try to mechanize whatever lends itself to such treatment.

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2. LOGICAL THEORY

a. PROPOSITIONAL CALCULI

Three of the well-known propositional calculi are formally presented here in the parentheses-free notation of Lukasiewicz:*

1. Classical Propositional Calculus
2. Fitch Propositional Calculus
3. Heyting Propositional Calculus

Logical systems that are extensions of these propositional calculi (i.e., functional calculi, class calculus, alethic logic, deontic logic, etc.) will be presented in later issues of MULL.

In the formulations of these propositional calculi the following are meta-linguistic formulas:

d d2 d3 ... e e2 e3 ...	meta-linguistic variables ranging over formulas in the object language
u u2 u3 ... v v2 v3 ... w w2 w3 ...	meta-linguistic variables ranging over well-formed formulas in the object language
* *‾	an abbreviation for the following: From: (the formula(s) written above the horizontal line) it is valid to infer: (the formula(s) written below the horizontal line)
* *	an abbreviation for: * *‾ where there is no formula, i.e., nothing, above the horizontal line

Thus, $\frac{*}{*}\ \frac{d}{e}$ will be read: From d it is valid to infer e.

Whereas, $\frac{*}{*}\ e$ will be read: From nothing it is valid to infer e (i.e., e is provable as a theorem).

The logical base of most formulations of any system are composed of an alphabet of signs, formation rules, axioms, transformation rules and definitions. In the formulations of the systems presented here the logical basis of each system will be associated with a name (called a FATD-number), which will determine the logical nature of that system. The FATD-number of a system will indicate just which ones on specified lists of Formation rules, Axioms, Transformation rules and Definitions are included in the corresponding logical basis of that system.

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*One of the incidental conveniences of the Lukasiewicz notation is that all of the characters used are available on the ordinary typewriter keyboard. An extension of the Lukasiewicz notation will be developed for the logical systems to be presented later so that they, too, can be presented by ordinary typewriter characters.

One other difference in the formulations presented here of the propositional calculi should be noted. These formulations do not contain any axioms. The equivalent effect of the axioms that are included in most formulations is achieved by the rules that are included in the formulations presented here. The following lists of formation rules, transformation rules and definitions will be used in formulating the three propositional calculi.

1. Formation Rules

Just as in any natural language there are certain combinations of signs that are meaningful and other combinations that are meaningless, there are similar combinations in these propositional calculi. In English, e.g., the combination of signs "dog" is a meaningful expression, but "bddst" is not. In these propositional calculi a combination of signs that is maaningful will be called a well-formed formula (abbreviated WFF). Criteria for determining whether a given candidate formula is well-formed in the various systems are given by various combinations of the following formation rules:

Rule Number	Statement of the Rule in Prose
FR1	If formula e is not a WFF by any of the following formation rules, then e is not a WFF.
FR2	If formula e is a propositional variable, then e is a WFF.
FR3	If formula e is a WFF, then Ne is a WFF.
FR4	If formulas d and e are WFF's, then Ade is a WFF.
FR5	If formulas d and e are WFF's, then Cde is a WFF.
FR6	If formulas d and e are WFF's, then Kde is a WFF.

2. Transformation Rules

In the transformation rules that follow, the capital letters used are intended to be interpreted in the following way:

Letter	Concept	Word(s) usually used to express the concept in English
C	material implication	if ..., then ...
N	negation	not
A	inclusive disjunction	or (in the sense of X or Y or both)
K	conjunction	and

With the exception of TR1, all of the rules used in the formulations of the propositional calculi presented here will be one or the other of two kinds of rules:

1. in rules — rules by which it is valid to infer from a set of statements that generally do not contain a given sign, a conclusion that does contain that given sign,

2. out rules — rules by which it is valid to infer from a set of statements that do contain a given sign, a conclusion that does not contain that given sign.

The names of the various transformation rules and their abbreviations are as follows:

R reiteration

Ki	conjunction-in	Ko	conjunction-out
Ci	implication-in	Co	implication-out
Ai	disjunction-in	Ao	disjunction-out
Ni	negation-in	No	negation-out
NNi	double-negation-in	NNo	double-negation-out
NAi	negation-disjunction-in	NAo	negation-disjunction-out
NKi	negation-conjunction-in	NKo	negation-conjunction-out

Rule Number	Rule Name	Statement of the Rule in Notation	Statement of the Rule in Prose
TR1	R	*_w * * w * * * w * * * * w ... etc.	From w, it is valid to infer from nothing 1. that w and 2. that from nothing it is valid to infer 1. that w and 2. that from nothing it is valid to infer 1. that w and 2. that from nothing it is valid to infer 1. that w and . . . etc. (If a proposition is true, then that proposition is true.)
TR2	Ki	* v *_w * Kvw	From v and w, it is valid to infer Kvw. (If two propositions are true, then the conjunction of them is true.)
TR3	Ko	*_Kvw * v * w	From Kvw, it is valid to infer v and it is valid to infer w. (If the conjunction of two propositions is true, then both of the propositions are true.)
TR4	Ci	* *_v * * : *_* w * Cvw	From the valid inference of w from v, it is valid to infer Cvw. (If one proposition can be validly inferred from a second proposition, then the second proposition implies the first.)

Rule Number	Rule Name	Statement of the Rule in Notation	Statement of the Rule in Prose
TR5	Co	* v *_Cvw * w	From v and Cvw, it is valid to infer w. (If one proposition is true and that proposition implies a second proposition, then that second proposition is true.)
TR6	Ai	*_v * Avw * Awv	From v, it is valid to infer Avw and it is valid to infer Awv. (If one proposition is true, then at least that proposition or a second proposition is true.)
TR7	Ao	* Auv * *_u * * : * * w * *_v * * : *_* w * w	From Auv and the valid inference of w from u and the valid inference of w from v, it is valid to infer w. (If at least one of two propositions is true and a third proposition can be validly inferred from the first proposition and also from the second proposition, then that third proposition is true.)
TR8	Ni	* *_v * * : * * w *_* Nw * Nv	From the valid inference of w and Nw from v, it is valid to infer Nv. (If a proposition leads to ridiculous results, namely a contradiction, then that proposition is not true.)
TR9	No	* v *_Nv * w	From v and Nv, it is valid to infer w. (If a contradiction is true, then any proposition is true.)
TR10	NNi	*_w * NNw	From w, it is valid to infer NNw. (If a proposition is true, than that proposition is not false.)
TR11	NNo	*_NNw * w	From NNw, it is valid to infer w. (If a proposition is not false, then that proposition is true.)
TR12	NAi	*_KNvNw * NAvw	From KNvNw, it is valid to infer NAvw. (If two propositions are false, then neither proposition is true.)
TR13	NAo	*_NAvw * KNvNw	From NAvw, it is valid to infer KNvNw. (If neither of two propositions is true, then both are false.)
TR14	NKi	*_ANvNw * KNvw	From ANvNw, it is valid to infer KNvw. (If at least one of two propositions is false, then it is not the case that both of them are true.)
TR15	NKo	*_NKvw * ANvNw	From NKvw, it is valid to infer ANvNw. (If it is not the case that both of two propositions are true, then at least one of them is false.)

3. Definitions

Definitions are used as shorthand devices. If a particular combination of signs occurs frequently enough, it becomes convenient to define a single sign that is used to represent that combination. Thus, the addition of definitions to a logical system makes that system more convenient to work with. They do not, however, change the logical nature of the system the way that the addition of some axioms or some transformation rules would. It should be noted that if a sign has already been introduced into a system by a transformation rule or an axiom, then that sign can no longer be introduced into the system by a definition.

Definition Number	Definition Name	Statement of the Definition in Notation
D1	Ad	Avw =df CNvw
D2	Kd	Avw =df NANvNw
D3	Ed	Evw =df KCvwCwv
D4	Jd	Jvw =df NEvw

The intended interpretation of E is as indicating equivalence, i.e., the concept usually expressed in English by "... if and only if ...". The intended interpretation of J is as indicating exclusive disjunction, i.e., the concept usually expressed in English by the word "or" when used in the sense of "X or Y but not both".

4. Classical Propositional Calculus (abbreviated CPC)

The logical basis of CPC, i.e., the things that are assumed, can be conveniently summarized as follows:

1. Alphabet

Primitive Connectives	C N
Propositional Variables	p p2 p3 ... q q2 q3 ... r r2 r3 ... s s2 s3 ...
2. Formation Rules	FR1 FR2 FR3 FR4
3. Transformation Rules	TR1 TR4 TR5 TR8 TR11
4. Definitions	D1 D2 D3 D4

The FATD-number associated with this formulation of CPC is F17-T2231-D17. The FATD-number of CPC, the numerical part of which is an octal number rather than a decimal number, indicates just which formation rules, axioms, transformation rules and definitions are included in the logical basis this formulation of CPC in the following manner.

This proof sets forth concisely -- but completely -- the reasoning that leads to the conclusion of MT1. Since we want to infer w from v and Nv, at Step 1 and Step 2 v and Nv are assumed as suppositions in the assertion column (the left column). These steps are justified by the letter s in the justification column (the right column), which indicates that the assertions made at these steps are suppositions. Our goal is to infer w. We know that if we can infer NNw, then we can infer w by the NNo rule. So the question now is: "Can we infer NNw?" We know that if the assumption of Nw leads to a contradiction, that we can then infer NNw by the Ni rule. So, at Step 3 we assume Nw to see whether it leads to a contradiction. At Step 4 v is asserted and is justified as a reiteration of Step 1. At Step 5 Nv is asserted and is justified as a reiteration of Step 2. Thus, the contradiction that both v and Nv are true can be inferred from the supposition of Nw. Therefore, at Step 6 NNw is asserted and is justified by the Ni rule because the assumption at Step 3 led to contradictory Steps 4 and 5. At Step 7 w is asserted and is justified by the application of the NNo rule to Step 6. Thus, from v and Nv at Steps 1 and 2, we have inferred by assumptions included in the logical basis of the formulation of CPC presented here, w at Step 7. So, the No rule is provable as MT1, i.e., as a meta-theorem about CPC.

Similarly, transformation rules 9, 10, 6, 7, 2, 3, 12, 13, 14 and 15, i.e., all of the rest of the transformation rules on the list, are also provable as meta-theorems about CPC. (See Appendix). Meta-theorems are different from transformation rules in one and only one important respect. Meta-theorems are provable, whereas transformation rules are assumed. However, once proved, the meta-theorems can be used to prove other theorems and meta-theorems in the same way that transformation rules are used.

An example of the proof of a theorem in CPC is the following:

T1 ANww

1* *_NNw	s
2* * w	1,NNo
3* CNNww	1-2,Ci
4* ANww	3,Ad,v/Nw

Notice in the justification part of Step 4 that v/Nw indicates that v has been replaced by Nw in the definition of disjunction.

Some other theorems provable in CPC are the following

T2	C KAvwNv w	T8	C Kvw v	T20	E Kww w
T3	C KCvwv w	T9	C v Avw	T21	E KuAvw AKuv Kuw
T4	C KCuvCvw Cuw	T10	E AAuvw AuAvw	T22	E Kvw Kwv
T5	C KCvwNw Nv	T11	E AKNvNwKvw Evw	T23	E NAvw KNvNw
T6	C KK ANvNw	T12	E ANvw Cvw	T24	E NKvw ANvNw
	Cu2v	T13	E Aww w	T25	E NNw w
	Cu3w	T14	E Avw Awv		
	ANu2Nu3	T15	E AuKvw KAuvAuw		
T7	C KK Au2u3	T16	E CKuvw CuCvw		
	Cu2v	T17	E CNvNw Cwv		
	Cu3w	T18	E Evw KCvwCwv		
	Avw	T19	E KKuvw KuKvw		

The F17 part of the CPC FATD-number indicates that formation rules 1, 2, 3 and 4 are in the logical basis of this formulation of CPC as follows:

1. Transform the octal number into a binary number:

Octal	Binary
17	001 111

2. Enumerate the binary number from right to left up to the number of rules that are included on the list of formation rules:

F 654 321

001 111

If the number of a rule is paired with a 1, then that formation rule is included in the logical basis of this formulation of CPC. If the number of a rule is paired with a 0, then that formation rule is not included in the logical basis of this formulation of CPC. Thus, the four 1's under 1, 2, 3 and 4 indicate that formation rules 1, 2, 3 and 4 are included in the logical basis of this formulation of CPC, while the 0's under 5 and 6 indicate that FR5 and FR6 are not.

Similarly, the T2231 part of the CPC FATD-number indicates that transformation rules 1, 4, 5, 8 and 11 are included in the logical basis of this formulation of CPC.

Octal	Binary
	T 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
2231	0 0 0 0 1 0 0 1 0 0 1 1 0 0 1

And in a similar manner, the D17 part indicates that definitions 1, 2, 3 and 4 are included in the logical basis of this formulation of CPC.

From this formulation there can be proved an infinite number of theorems in CPC and a similar number of meta-theorems about CPC. Among the meta-theorems that can be proved is TR9, the No rule. The proof of TR9 is set forth below as meta-theorem number one to illustrate a formal proof in this formulation of CPC.

MT1 From v and Nv, it is valid to infer w

1* v	s
2* Nv	s
3* * Nw	s
4* * v	1R
5* * Nv	2R
6* NNw	3-4-5,Ni
7* w	6,NNo

Notice that if (1) all instances of one or more consecutive occurrences of propositional variables in these 25 theorems are replaced by the place-marker ' and the resulting expressions are called theorem-patterns and (2) the place-marker ' is stipulated as succeeding all the capital letters in alphabetic order, then (3) the 25 theorems are arranged in the alphabetic order of their respective theorem-patterns.

T1 AN'
T2 CK A'N'
T3 CK C'
T4 CK C'C' C'
T5 CK C'N' N'

T6 CK KA N'N' C'C' AN' N'
T7 CK KA' C'C' A'
T8 CK'
T9 C'A'
T10 EA A'A' A'

T11 EA KN' N'K' E'
T12 EA N'C'
T13 EA'
T14 EA' A'
T15 EA' K'K A'A'

T16 EC K'C' C'
T17 EC N'N' C'
T18 EE' KC' C'
T19 EK K'K' K'
T20 EK'

T21 EK' A'A K'K'
T22 EK' K'
T23 EN A'K N'N'
T24 EN K'A N'N'
T25 EN N'

This device of replacing propositional variables by a place-marker thus provides a method for indexing the theorems in CPC. When the Lukasiewicz notation is extended, there will be a similar method for indexing the meta-theorems about CPC.

Logicians have been interested in and have investigated various fragments of CPC because it turns out that certain theorems provable in CPC are somewhat paradoxical. Two of the more interesting fragments of CPC that have been intensively investigated are FPC and HPC.

5. Fitch Propositional Calculus (abbreviated FPC)

The assumptions in the logical basis of one formulation of FPC can be conveniently summarized in the following:

1. Alphabet	The same as for CPC with the addition of A and K as primitive connectives
2. Formation Rules	FR1 FR2 FR3 FR4 FR5 FR6
3. Transformation Rules	TR1 TR2 TR3 TR4 TR5 TR6 TR7 TR9 TR10 TR11 TR12 TR13 TR14 TR15
4. Definitions	D3 D4

The FATD-number associated with this formulation of FPC is F77-T77577-D14. The F77 part indicates the formation rules included in the logical basis of this formulation of FPC.

Octal	Binary
	F 6 5 4 3 2 1
77	1 1 1 1 1 1

The T77577 part indicates the transformation rules included in the logical basis of this formulation of FPC.

Octal	Binary
	T 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
77577	1 1 1 1 1 1 1 0 1 1 1 1 1 1 1

The D14 part indicates the definitions included in the logical basis of this formulation of FPC.

Octal	Binary
	D 4 3 2 1
14	1 1 0 0

It should be noted that TR8 is not provable as a meta-theorem about FPC. Thus, FPC is a weaker system than CPC, e.g., there are meta-linguistic statements about CPC that are either assumed or provable such that the corresponding statements about FPC are neither assumed nor provable. There are also formulas that are provable as theorems in CPC, but which are not provable as theorems in FPC. One example of such an expression is ANww. Thus, if system S1 is weaker than system S2, then S1 is a sub-system of S2, i.e., all the formulas that are provable as theorems in S1 are also provable as theorems in S2, but not vice-versa.

6. Heyting Propositional Calculus (abbreviated HPC)

The assumptions in the logical basis of this formulation of HPC can be conveniently summarized as follows:

1. Alphabet	The same as for FPC
2. Formation Rules	FR1 FR2 FR3 FR4 FR5 FR6
3. Transformation Rules	TR1 TR2 TR3 TR4 TR5 TR6 TR7 TR8 TR9
4. Definitions	D3 D4

The FATD-number associated with this formulation of HPC is F77-T777-D14. It can be ascertained that HPC is a sub-system of CPC. However, HPC is not a sub-system of FPC nor is FPC a sub-system of HPC.

It should be noted that there are other formulations of these three propositional calculi than the formulations presented here. For example, the system associated with the FATD-number F77-T2377-D14 is another formulation of CPC.

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Appendix

TR10 NNi

MT2 From w, it is valid to infer NNw

1*	_w	s
2*	* _Nw	s
3*	* * w	1R
4*	NNw	2-3,Ni

TR6 Ai

MT3 From v, it is valid to infer Avw and it is valid to infer Awv

1*	_v	s
2*	* _Nv	s
3*	* * v	1R
4*	* w	2-3,No (MT1)
5*	CNvw	2-4,Ci
6*	* _Nw	s
7*	* * v	1R
8*	CNwv	6-7,Ci
9*	Avw	5,Ad
10*	Awv	8,Ad

TR7 Ao

MT4 From Auv and the valid inference of w from u and the valid inference of w from v, it is valid to infer w

1*	Auv	s
2*	* _u	s
	* * :	
3*	* w	s
4*	* _v	s
	* * :	
5*	_* w	s
6*	* _Nw	s
7*	* * u	s
8*	* * Cuw	2-3,Ci
9*	* * w	7,8,Co
10*	* * Nw	6R
11*	* Nu	7-9-10,Ni
12*	* Nv	Similar to 6-11
13*	* CNuv	1R,Ad
14*	* v	11,13,Co
15*	NNw	6-12-14,Ni
16*	w	15,NNo

TR2 Ki

MT5 From v and w, it is valid to infer Kvw

1*	v	s
2*	_w	s
3*	* _NKvw	s
4*	* NNANvNw	3,Kd
5*	* ANvNw	4,NNo
6*	* CNNvNw	5,Ad
7*	* NNv	1R,NNi(MT2)
8*	* Nw	6,7,Co
9*	* w	2R
10*	Kvw	3-8-9,Ni,NNo

TR3 Ko

MT6 From Kvw, it is valid to infer v and it is valid to infer w

1*	_Kvw	s
2*	* _Nv	s
3*	* * _NNv	s
4*	* * * Nv	2R
5*	* * * Nw	3,4,No(MT1)
6*	* CNNvNw	3-5,Ci
7*	* ANvNw	6,Ad
8*	* NANvNw	1R,Kd
9*	v	2-7-8,Ni,NNo
10*	w	Similar to 2-9

TR12 NAi

MT7 From KNvNw, it is valid to infer NAvw

1*	_KNvNw	s
2*	* _Avw	s
3*	* CNvw	2,Ad
4*	* Nv	1R,Ko(MT6)
5*	* w	3,4,Co
6*	* Nw	1R,Ko(MT6)
7*	NAvw	2-5-6,Ni

TR13 NAO

MT8 From NAvw, it is valid to infer KNvNw

1*	NAvw	s
2*	* NKNvNw	s
3*	* NNANNvNNw	2,Kd
4*	*ANNvNNw	3,NNo
5*	* * Nv	s
6*	* * CNNNvNNw	4R,Ad
7*	* * NNNv	5,NNi(MT2)
8*	* * NNw	6,7,Co
9*	* * w	8,NNo
10*	* CNvw	5-9,Ci
11*	* Avw	10,Ad
12*	* NAvw	1R
13*	KNvNw	2-11-12,Ni,NNo

TR14 NKi

MT9 From ANvNw, it is valid to infer NKvw.

1*	ANvNw	s
2*	* Kvw	s
3*	* NANvNw	2,Kd
4*	* ANvNw	1R
5*	NKvw	2-3-4,Ni

TR15 NKo

MT10 From NKvw, it is valid to infer ANvNw

1*	NKvw	s
2*	NNANvNw	1,Kd
3*	ANvNw	2,NNo

3. REVIEWS AND ABSTRACTS

J U R I M E T R I C S

The basic importance of perception to the legal process needs little emphasis to lawyers familiar with trials and the rules of evidence . There have long been voices warning that testimonial evidence was not necessarily all it was assumed to be. Now Science is beginning to supply us with specifications as to the process of perception, what it is, and what it is not.

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a. Visual Perception and Personality, by Warren J. Wittreich, Scientific American, April, 1959, p. 56.

Perception obviously involves intellectual assessment of stimuli as when the shrinking image of a receding figure is interpreted to indicate a difference in distance rather than in size. Recent experiments show that the emotions, too, are involved in perception, not only of such intangibles as beauty, but also of such "objective" attributes as stature.

The experiments reported involve the observation of individuals in the situation of the "Ames room" by others having a particular relationship to them. The "Ames room" is a room devised by lawyer-turned-psychologist Adelbert Ames, Jr., in which the physical characteristics and dimensions of the room are distorted from normal in such fashion as to produce surprising aberrations of perception. Thus, depending upon which direction an individual walks in the Ames room he appears to an observer to grow or shrink.

In the experiments reported, wives observed husbands, enlisted men observed their military superiors, and amputees observed other amputees. In each case the observer reported less apparent distortion of the emotionally-related subject in the Ames room situation than of an observed neutral subject. It is the hypothesis of the experimenters that an anxiety-inducing figure is less easily distorted in the perceptive process than a neutralfigure. In any event, the experiments seem to demonstrate that our reactions to people and our perceptions of them are not separate processes, but merely aspects of a single process, and that feeling and perceiving are deeply fused and complexly interrelated in the process of living.

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b. Experiments in Color Vision, by Edwin H. Land, Scientific American, May, 1959, p. 84.

The classical theory of color vision, based upon the wave motion of light, is that the order of colors in the spectrum follows the wavelength, with the longest visible wavelength falling at the red end of the spectrum and the shortest at the violet end. In mixing spectral stimuli, it has been found that three different wavelengths chosen from the red, green and blue bands of the spectrum were enough to match all colors. Accordingly, red, green and blue came to be called primary colors. Scientific theory up to the present has held that the color seen at any point in a field of view depends on the wavelengths issuing from that point and on their relative strengths or intensities.

It is now demonstrated by an ingenious series of experiments that color perception does not arise from the wavelengths issuing from a particular point but from the interplay of longer and shorter wavelengths over the entire field. Thus a colored scene is photographed through two different filters, producing negatives that record slightly longer and shorter wavelengths emanating from the scene. These negatives are then used in projectors illuminated by light sources of longer and shorter wavelengths. It is found that a full spectrum of colors may be thus produced even from light sources that supply only light from two narrow ranges of wavelength. It is thus demonstrated that color in the natural image depends on the random interplay of longer and shorter wavelengths over the total visual field and not upon the wavelength of the light emanating from a particular point or part of the field.

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c. The Perception of Motion, by Hans Wallach, Scientific American, July 1959, p. 56.

Most people have experienced the illusion of motion when on a train and when the motion of an adjacent train creates a feeling of the observer's motion. Similarly, the moon observed through racing clouds seems itself to be moving rapidly across the sky.

Experimental investigation of these and similar illusions with respect to the perception of motion has been undertaken by recording the perception of motion reported by various observers in controlled situations with dots, squares and circles placed in various positions relative to each other. Certain principles seem to be established. A surrounded object appears to move in relation to its surrounding object or field, even though the latter is, in fact, moving relative to the observer at a perceptible velocity and the surrounded object is at rest relative to the observer. Also the velocity of an object observed through an aperture seems to be greater if the aperture is smaller. The velocity reported as perceived by observers seems to vary inversely to the size of the field against which the observation is made. Thus, to the extent that the awareness of motion depends upon visual perception it is subject to vagaries demonstrated in this group of experiments.

It has, of course, been known to many for years that perception is subject to vagaries of various kinds. However, it is only relatively recently that science has begun to provide accurate insight as to the character of perception and its aberrations. There are now numerous reports in the field, in addition to the three recent ones abstracted above. Anthropological studies have indicated, for example, that the division of the color spectrum into the colors normally "perceived" is an arbitrary matter, dependent upon the cultural background of the perceiver. There is no "natural" division of colors. The colors that one sees are those that the culture has taught one to see and name. (Scientific American, Oct. 1954, p. 50). Series of experiments, including some astonishing illusions produced by the "Ames room", demonstrate clearly that what is perceived is not a direct and automatic response to some external stimulus or "reality", but rather is a judgment, based upon a weighted average of our own past experience in interpreting the stimuli that come to us through our senses. (Scientific American, August 1951, p. 50).

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Note that the pairs of capital letters at the end of each entry indicate that the document contains information about one or more of the following categories:

DI Drafting and Interpreting usage of modern logic in law
DL Deontic Logic
GD General Discussion of law and logic
GT General Textbook in symbolic logic
HS Hohfeld's System of Analysis
IR Information Retrieval: law and logic
LS Materials used in the SYMBOLIC LOGIC AND LEGAL COMMUNICATION seminar at Yale Law School in 1958

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1. Kraszewski, Z. 23 JSL 73 (1958).

5. PROBLEMS

a. Reed Lawlor sent in the following problem published recently in another periodical (citation to be given in next issue):

> If the policeman testifies that anything which the prisoner deposes is false, and the prisoner deposes that something which the policeman testifies is true, then what conclusions can be validly deduced?

Anyone care to send in responses for publication in the next issue?

6. READERS' LETTERS AND OTHER ITEMS ABOUT MULL

a. Subscriptions to MULL

"Unfortunately the system for financing the newsletter is not well adapted to government operations which would be free to enter a subscription but could not very well use the voluntary contribution device.

"Would it be possible to establish a subscription rate on a more formal basis for those organizations which are similarly handicapped in financial accountability procedures?"

D. D. Andrews

(Henceforth, MULL will be available to institutional subscribers at the rate of $2 for one year, $5 for three years. To individuals, MULL will continue to be available on a voluntary contribution basis. --ED.)

b. Contents of MULL

"One suggestion that I have for information that might be included in such a newsletter is the possibility of publishing abstracts of articles appearing in scientific journals and containing scientific information that may be of significance to lawyers and the law. One example that I think of at the moment is an article that appeared in SCIENTIFIC AMERICAN within the past two or three months. This article reported on the effect of emotional attitudes on observation. It was based upon the classical experiment that has been conducted, I believe at Princeton, in which distances and sizes are distorted in apparent perception by virtue of false clues built into the environment. The article I have in mind reported a significant differential in the error of perception involved when various subjects viewed other persons, depending upon the emotional relationship between the subject and the person viewed. It seems to me that the significance of such an experiment for the evaluation of testimony and other legal purposes is obvious and great."

Lee Loevinger

(See 59S3. --ED.)

c. Citations to MULL

To keep citations brief, the following system will be used in making citations to MULL. Example: MULL 59S6c18

59	Volume number -- coincides with year of publication	
S	Month of publication --	M March
		J June
		S September
		D December
6	Section number	
c	Subsection number	
18	Page number (if necessary)	

When the item being referred to appears in the same issue of MULL as does the citation, then the name of the newsletter is omitted. Thus, 59S6c is the citation for this section.

d. Suggested Topic for Discussion

"I believe it would be helpful, as a suggested topic for an article or news item, to have a discussion of the utilization of symbolic logic and electronics as applied to statutory law in contrast with common law. I refer expressly to a fairly complicated statute with a considerable volume of administrative rulings and case litigation such as the Internal Revenue Code."

Keith E. Morrison

(Anybody wish to tackle this one for the next issue? -- ED.)

e. Translating by Machine

"This piece in the Manshester Guardian (June 18, 1959 entitled TRANSLATING BY MACHINE) might serve as the basis of an item in your newsletter. For my own part, I consider legal research by machine closely related to translating by machine...

"As to the idea that language can only be understood when it is being used to communicate something, it is not new to the law. See L. Hand, J., in NLRB v. Federbush Co. (CA 2, 1941) 121 F.2d 954, 957:

> Words are not just pebbles in alien juxtaposition; they have only a communal existence; and not only does the meaning of each interpenetrate the other, but all in their aggregate take their purpose from the setting in which they are used, of which the relation between the speaker and the hearer is perhaps the most important part.

Or, as another great master of language, Matthew Arnold said:

> There is always a danger in quoting out of context, since the whole of a work is what produces its effect, not a particular paragraph, sentence or line."

J. Raymond Dyer

(Any takers to discuss the relationship between translation by machine and legal research? --ED.)

f. Western Reserve Conference on Machine Retrieval of Information

An international Conference for Standards on A Common Language For Machine Searching and Translation, sponsored by Western Reserve University and the Rand Development Corporation was held September 6-12, 1959 in Cleveland. International interest was evidenced by the presentation of 52 papers from 10 countries at the conference. Senator Hubert H. Humphrey of Minnesota delivered the keynote address at a dinner on September 9.

g. Symbolic Logic in Legal Education

A review by Zdzislaw Draszewski in 23 JSL 73 (1958) notes an interesting development in post World War II legal education in Poland:

> " ... courses in logic are obligatory for students of law at all Polish Universities."

The four books reviewed, all designed to teach logic to law students, were written by lawyers, not professional logicians.

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*   Since one of the major purposes of MULL is to provide       *
*   a vehicle for the exchange of information about             *
*                                                               *
*        1. the uses of modern logic in law and                 *
*                                                               *
*        2. modern methods of information retrieval in          *
*           law,                                                *
*                                                               *
*   readers are urged to send in news about research in         *
*   progress that will be appropriate for inclusion in          *
*   MULL.                                                       *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
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